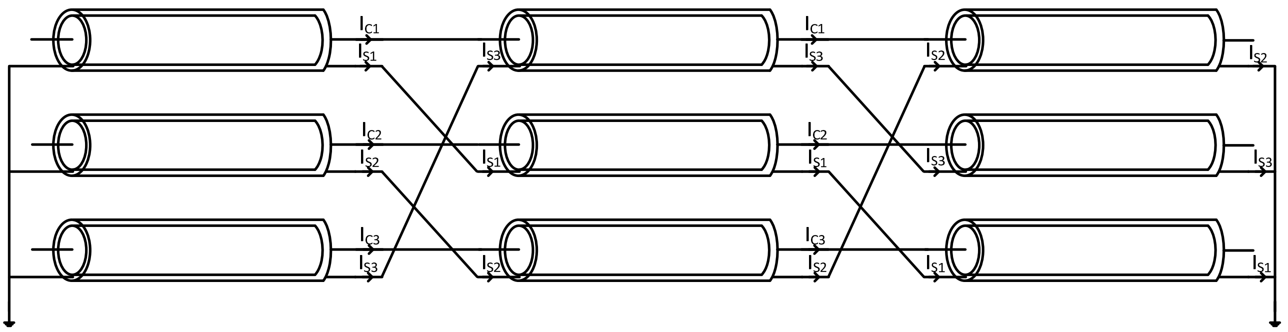
Chapter III

1)

We are going to calculate the series impedance and admittance for each minor-section of the figure below.



The series impedance matrix per unit of length of the first minor-section is identical to the one of the cable bonded in both-ends.

$$
\begin{array}{cc}
 & \begin{array}{cccccc} \textit{Core 1} & \textit{Screen 1} & \textit{Core 2} & \textit{Screen 2} & \textit{Core 3} & \textit{Screen 3} \end{array} \\
[Z_1]= & \begin{bmatrix}
0.0761+j0.688 & 0.0494+j0.629 & 0.0494+j0.577 & 0.0494+j0.577 & 0.0494+j0.577 & 0.0494+j0.577 \\
0.0494+j0.629 & 0.2284+j0.628 & 0.0494+j0.577 & 0.0494+j0.577 & 0.0494+j0.577 & 0.0494+j0.577 \\
0.0494+j0.577 & 0.0494+j0.577 & 0.0761+j0.688 & 0.0494+j0.629 & 0.0494+j0.577 & 0.0494+j0.577 \\
0.0494+j0.577 & 0.0494+j0.577 & 0.0494+j0.629 & 0.2284+j0.628 & 0.0494+j0.577 & 0.0494+j0.577 \\
0.0494+j0.577 & 0.0494+j0.577 & 0.0494+j0.577 & 0.0494+j0.577 & 0.0761+j0.688 & 0.0494+j0.629 \\
0.0494+j0.577 & 0.0494+j0.577 & 0.0494+j0.577 & 0.0494+j0.577 & 0.0494+j0.629 & 0.2284+j0.628
\end{bmatrix} m\Omega / m
\end{array}
$$

For the second minor-section there is a shifting of the screens resulting in the series impedance matrix shown below. The mutual coupling from screen 1 to the other screens and conductors is now where there was before screen 2, whereas screen 2 is now where there was before screen 3, whereas screen 3 is now where there was before screen 1.

$$
\begin{array}{cc}
 & \begin{array}{cccccc} \textit{Core 1} & \textit{Screen 3} & \textit{Core 2} & \textit{Screen 1} & \textit{Core 3} & \textit{Screen 2} \end{array} \\
[Z_2]= & \begin{bmatrix}
0.0761+j0.688 & 0.0494+j0.577 & 0.0494+j0.577 & 0.0494+j0.629 & 0.0494+j0.577 & 0.0494+j0.577 \\
0.0494+j0.577 & 0.2284+j0.628 & 0.0494+j0.577 & 0.0494+j0.577 & 0.0494+j0.629 & 0.0494+j0.577 \\
0.0494+j0.577 & 0.0494+j0.577 & 0.0761+j0.688 & 0.0494+j0.577 & 0.0494+j0.577 & 0.0494+j0.629 \\
0.0494+j0.629 & 0.0494+j0.577 & 0.0494+j0.577 & 0.2284+j0.628 & 0.0494+j0.577 & 0.0494+j0.577 \\
0.0494+j0.577 & 0.0494+j0.629 & 0.0494+j0.577 & 0.0494+j0.577 & 0.0761+j0.688 & 0.0494+j0.577 \\
0.0494+j0.577 & 0.0494+j0.577 & 0.0494+j0.629 & 0.0494+j0.577 & 0.0494+j0.577 & 0.2284+j0.628
\end{bmatrix} m\Omega / m
\end{array}
$$

For the third minor-section there is again a shifting on the screens' mutual coupling.

$$\begin{array}{rc}
 & \begin{array}{cccccc} \textit{Core 1} & \textit{Screen 2} & \textit{Core 2} & \textit{Screen 3} & \textit{Core 3} & \textit{Screen 1} \end{array} \\
[Z_3]= & \begin{bmatrix}
0.0761+j0.688 & 0.0494+j0.577 & 0.0494+j0.577 & 0.0494+j0.577 & 0.0494+j0.577 & 0.0494+j0.629 \\
0.0494+j0.577 & 0.2284+j0.628 & 0.0494+j0.629 & 0.0494+j0.577 & 0.0494+j0.577 & 0.0494+j0.577 \\
0.0494+j0.577 & 0.0494+j0.629 & 0.0761+j0.688 & 0.0494+j0.577 & 0.0494+j0.577 & 0.0494+j0.577 \\
0.0494+j0.577 & 0.0494+j0.577 & 0.0494+j0.577 & 0.2284+j0.628 & 0.0494+j0.629 & 0.0494+j0.577 \\
0.0494+j0.577 & 0.0494+j0.577 & 0.0494+j0.577 & 0.0494+j0.629 & 0.0761+j0.688 & 0.0494+j0.577 \\
0.0494+j0.629 & 0.0494+j0.577 & 0.0494+j0.577 & 0.0494+j0.577 & 0.0494+j0.577 & 0.2284+j0.628
\end{bmatrix} m\Omega/m
\end{array}$$

As we consider an ideal cross-bonding and that the minor-sections have equal length, the series impedance of the cable is given by the average of the three minor-sections.

The two previous conditions are very important. To have minor-sections with different lengths could be handled by weighting the parameters when calculating the average, but if we not consider ideal cross-bonding we would have to consider the output voltage and current of one minor-section as the input voltage and current of the next minor-section, which would make the calculations more difficult.

$$[Z_T]=\begin{bmatrix}
0.0761+j0.688 & 0.0494+j0.595 & 0.0494+j0.577 & 0.0494+j0.595 & 0.0494+j0.577 & 0.0494+j0.595 \\
0.0494+j0.595 & 0.2284+j0.628 & 0.0494+j0.595 & 0.0494+j0.577 & 0.0494+j0.595 & 0.0494+j0.577 \\
0.0494+j0.577 & 0.0494+j0.577 & 0.0761+j0.688 & 0.0494+j0.595 & 0.0494+j0.577 & 0.0494+j0.595 \\
0.0494+j0.595 & 0.0494+j0.595 & 0.0494+j0.595 & 0.2284+j0.628 & 0.0494+j0.595 & 0.0494+j0.577 \\
0.0494+j0.577 & 0.0494+j0.577 & 0.0494+j0.577 & 0.0494+j0.595 & 0.0761+j0.688 & 0.0494+j0.595 \\
0.0494+j0.595 & 0.0494+j0.595 & 0.0494+j0.595 & 0.0494+j0.577 & 0.0494+j0.595 & 0.2284+j0.628
\end{bmatrix} m\Omega/m$$

The admittance matrix is calculated exactly on the same way than the series impedance matrix.

$$[Y_1]=\begin{bmatrix}
j0.0761 & -j0.0761 & 0 & 0 & 0 & 0 \\
-j0.0761 & j0.4595 & 0 & 0 & 0 & 0 \\
0 & 0 & j0.0761 & -j0.0761 & 0 & 0 \\
0 & 0 & -j0.0761 & j0.4595 & 0 & 0 \\
0 & 0 & 0 & 0 & j0.0761 & -j0.0761 \\
0 & 0 & 0 & 0 & -j0.0761 & j0.4595
\end{bmatrix} \mu S/m$$

$$[Y_2]=\begin{bmatrix}
j0.0761 & 0 & 0 & -j0.0761 & 0 & 0 \\
0 & j0.4595 & 0 & 0 & -j0.0761 & 0 \\
0 & 0 & j0.0761 & 0 & 0 & -j0.0761 \\
-j0.0761 & 0 & 0 & j0.4595 & 0 & 0 \\
0 & -j0.0761 & 0 & 0 & j0.0761 & 0 \\
0 & 0 & -j0.0761 & 0 & 0 & j0.4595
\end{bmatrix} \mu S/m$$

$$[Y_1]=\begin{bmatrix}
j0.0761 & 0 & 0 & 0 & 0 & -j0.0761 \\
0 & j0.4595 & -j0.0761 & 0 & 0 & 0 \\
0 & -j0.0761 & j0.0761 & 0 & 0 & 0 \\
0 & 0 & 0 & j0.4595 & -j0.0761 & 0 \\
0 & 0 & 0 & -j0.0761 & j0.0761 & 0 \\
-j0.0761 & 0 & 0 & 0 & 0 & j0.4595
\end{bmatrix} \mu S/m$$

$$\left[Y_T\right]=\begin{bmatrix} j0.0761 & -j0.0260 & 0 & -j0.0260 & 0 & -j0.0260 \\ -j0.0260 & j0.4595 & -j0.0260 & 0 & -j0.0260 & 0 \\ 0 & -j0.0260 & j0.0761 & -j0.0260 & 0 & -j0.0260 \\ -j0.0260 & 0 & -j0.0260 & j0.4595 & -j0.0260 & 0 \\ 0 & -j0.0260 & 0 & -j0.0260 & j0.0761 & -j0.0260 \\ -j0.0260 & 0 & -j0.0260 & 0 & -j0.0260 & j0.4595 \end{bmatrix} \mu S / m$$

2)
We already have the values for the different entries of the loop matrix, from example III.3.1.1:
Z_{Couter}=2.6687x10^{-5}+j1.468x10^{-5} Ω/m
$Z_{CSinsul}$=0+j4.256x10^{-5} Ω/m
Z_{Sinner}=1.790x10^{-4}+j9.791x10^{-7} Ω/m
Z_{Souter}=1.7790x10^{-4}+j9.353x10^{-7} Ω/m
$Z_{SGinsul}$=0+j6.605x10^{-6} Ω/m
$Z_{Smutual}$=1.790x10^{-4}-j4.784x10^{-8} Ω/m
Z_{earth}≈4.945x10^{-5}+j6.209x10^{-4} Ω/m
Z_{earth_mutual}≈4.945x10^{-5}+j5.774x10^{-4} Ω/m

The loop matrix is given as shown below.

$$[Z_L]=\begin{bmatrix} Z_{L\_CS} & Z_{Lm\_CS} & 0 & 0 & 0 & 0 \\ Z_{Lm\_CS} & Z_{L\_SG} & 0 & Z_{gm\_12} & 0 & Z_{gm\_13} \\ 0 & 0 & Z_{L\_CS} & Z_{Lm\_CS} & 0 & 0 \\ 0 & Z_{gm\_12} & Z_{Lm\_CS} & Z_{L\_SG} & 0 & Z_{gm\_23} \\ 0 & 0 & 0 & 0 & Z_{L\_CS} & Z_{Lm\_CS} \\ 0 & Z_{gm\_13} & 0 & Z_{gm\_23} & Z_{Lm\_CS} & Z_{L\_SG} \end{bmatrix}$$

The calculation of the different terms of the loops matrix is direct:

$$Z_{L\_CS}=Z_{Couter}+Z_{CSinsul}+Z_{Sinner}$$
$$\Leftrightarrow Z_{L\_CS}=\left(2.6687x10^{-5}+j1.468x10^{-5}\right)+\left(0+j4.256x10^{-5}\right)+\left(1.790x10^{-4}+j9.791x10^{-7}\right)$$
$$\Leftrightarrow Z_{L\_CS}=2.057x10^{-4}+j5.822x10^{-5}\ \ \Omega/m$$

$$Z_{L\_SG}=Z_{Souter}+Z_{SGinsul}+Z_{earth}$$
$$\Leftrightarrow Z_{L\_SG}=\left(1.779x10^{-4}+j9.353x10^{-7}\right)+\left(0+j6.605x10^{-6}\right)+\left(4.945x10^{-5}+j6.209x10^{-4}\right)$$
$$\Leftrightarrow Z_{L\_SG}=2.274x10^{-4}+j6.284x10^{-4}\ \ \Omega/m$$

$$Z_{Lm\_CS}=-Z_{Smutual}\Leftrightarrow Z_{Lm\_CS}=-1.790x10^{-4}+j4.784x10^{-8}\ \ \Omega/m$$

$$Z_{gmij}=4.945x10^{-5}+j5.774x10^{-4}\ \ \Omega/m$$

Replacing the values on the matrix, it is obtained:

$$[Z_L]=\begin{bmatrix} 2.057x10^{-4}+j5.822x10^{-5} & 1.790x10^{-4}-j4.784x10^{-8} & 0 & 0 & 0 & 0 \\ 1.790x10^{-4}-j4.784x10^{-8} & 2.274x10^{-4}+j6.284x10^{-4} & 0 & 4.945x10^{-5}+j5.774x10^{-4} & 0 & 4.945x10^{-5}+j5.774x10^{-4} \\ 0 & 0 & 2.057x10^{-4}+j5.822x10^{-5} & 1.790x10^{-4}-j4.784x10^{-8} & 0 & 0 \\ 0 & 4.945x10^{-5}+j5.774x10^{-4} & 1.790x10^{-4}-j4.784x10^{-8} & 2.274x10^{-4}+j6.284x10^{-4} & 0 & 4.945x10^{-5}+j5.774x10^{-4} \\ 0 & 0 & 0 & 0 & 2.057x10^{-4}+j5.822x10^{-5} & 1.790x10^{-4}-j4.784x10^{-8} \\ 0 & 4.945x10^{-5}+j5.774x10^{-4} & 0 & 4.945x10^{-5}+j5.774x10^{-4} & 1.790x10^{-4}-j4.784x10^{-8} & 2.274x10^{-4}+j6.284x10^{-4} \end{bmatrix}$$

Before calculating the series impedance matrix, we need to know the transformation matrix.

$$[Z]=[A]^{-T}[Z_L][A]^{-1}$$

$$[A]=\begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ -1 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & -1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & -1 & 1 \end{bmatrix}$$

Now, we are ready to calculate the series impedance matrix:

$$[Z]=\begin{bmatrix} 0.0751+j0.687 & 0.0484+j0.628 & 0.0495+j0.577 & 0.0495+j0.577 & 0.0495+j0.577 & 0.0495+j0.577 \\ 0.0484+j0.628 & 0.2274+j0.628 & 0.0495+j0.577 & 0.0495+j0.577 & 0.0495+j0.577 & 0.0495+j0.577 \\ 0.0495+j0.577 & 0.0495+j0.577 & 0.0751+j0.687 & 0.0484+j0.628 & 0.0495+j0.577 & 0.0495+j0.577 \\ 0.0495+j0.577 & 0.0495+j0.577 & 0.0484+j0.628 & 0.2274+j0.628 & 0.0495+j0.577 & 0.0495+j0.577 \\ 0.0495+j0.577 & 0.0495+j0.577 & 0.0495+j0.577 & 0.0495+j0.577 & 0.0751+j0.687 & 0.0484+j0.628 \\ 0.0495+j0.577 & 0.0495+j0.577 & 0.0495+j0.577 & 0.0495+j0.577 & 0.0484+j0.628 & 0.224+j0.628 \end{bmatrix} m\Omega / m$$

There are minor differences between this matrix and the one shown in example III.3.1.1, because of the approximations done in this calculation.

3)

In this example it is added an extra conductive layer to each phase.

Parts of impedances used in the calculation of the loop impedance are identical to the ones of the previous exercise, as the expressions are not affects by the adding of an extra phase:

Z_{Couter}=2.6687x10^{-5}+j1.468x10^{-5} Ω/m
$Z_{CSinsul}$=0+j4.256x10^{-5} Ω/m
Z_{Sinner}=1.790x10^{-4}+j9.791x10^{-7} Ω/m
Z_{Souter}=1.7790x10^{-4}+j9.353x10^{-7} Ω/m
$Z_{SAinsul}$=0+j6.605x10^{-6} Ω/m
$Z_{Smutual}$=1.790x10^{-4}-j4.784x10^{-8} Ω/m

The first step is to calculate the penetration depths for the armour and earth:

$$m_A = \sqrt{\frac{j\omega\mu_A}{\rho_A}} \Leftrightarrow m_A = \sqrt{\frac{j2\pi\cdot 50\cdot 1000\cdot 4\pi\times 10^{-7}}{9.1\times 10^{-7}}} \Leftrightarrow m_A = 466 + j466\ m^{-1}$$

$$m_e = \sqrt{\frac{j\omega\mu_e}{\rho_e}} \Leftrightarrow m_e = \sqrt{\frac{j2\pi\cdot 50\cdot 1\cdot 4\pi\times 10^{-7}}{100}} \Leftrightarrow m_A = 1.4\times 10^{-3} + j1.4\times 10^{-3}\ m^{-1}$$

We can now calculate the remaining impedances. We star by calculating the inner series impedance of the armour:

$$Z_{Ainner} = \frac{\rho_A m_A}{2\pi R_4}\coth\left(m_A\left(R_5 - R_4\right)\right) - \frac{\rho_A}{2\pi R_4\left(R_4 + R_5\right)}$$

$$\Leftrightarrow Z_{Ainner} = \frac{9.1\times 10^{-7}\left(466 + j466\right)}{2\pi\cdot 47.5\times 10^{-3}}\coth\left(\left(466 + j466\right)\left(52.5 - 47.5\right)\times 10^{-3}\right) - \frac{9.1\times 10^{-7}}{2\pi 47.5\times 10^{-3}\left(52.5 + 47.5\right)\times 10^{-3}}$$

$$\Leftrightarrow Z_{Ainner} = 1.36\times 10^{-3} + j1.45\times 10^{-3}\ \Omega / m$$

Outer series impedance of the armour:

$$Z_{Aouter} = \frac{\rho_A m_A}{2\pi R_5}\coth\left(m_A\left(R_5 - R_4\right)\right) + \frac{\rho_A}{2\pi R_5\left(R_4 + R_5\right)}$$

$$\Leftrightarrow Z_{Aouter} = \frac{9.1\times 10^{-7}\left(466 + j466\right)}{2\pi\cdot 52.5\times 10^{-3}}\coth\left(\left(466 + j466\right)\left(52.5 - 47.5\right)\times 10^{-3}\right) - \frac{9.1\times 10^{-7}}{2\pi 52.5\times 10^{-3}\left(52.5 + 47.5\right)\times 10^{-3}}$$

$$\Leftrightarrow Z_{Aouter} = 1.29\times 10^{-3} + j1.31\times 10^{-3}\ \Omega / m$$

Series impedance of the armour insulation:

$$Z_{AGinsul} = \frac{j\omega\mu_{out2\_ins}}{2\pi}\ln\left(\frac{R_6}{R_5}\right) \Leftrightarrow Z_{AGinsul} = \frac{j2\pi 50\cdot 4\pi\times 10^{-7}}{2\pi}\ln\left(\frac{54.5}{52.5}\right) \Leftrightarrow Z_{AGinsul} = j2.349\times 10^{-6}\ \Omega / m$$

Mutual series impedance for the armour:

$$Z_{Amutual} = \frac{\rho_A m_A}{\pi\left(R_4 + R_5\right)}\operatorname{csch}\left(m_A\left(R_5 - R_4\right)\right)$$

$$\Leftrightarrow Z_{Amutual} = \frac{9.1\times 10^{-7}\left(466 + j466\right)}{\pi\left(52.5 + 47.5\right)\times 10^{-3}}\operatorname{csch}\left(\left(466 + j466\right)\left(52.5 - 47.5\right)\times 10^{-3}\right)$$

$$\Leftrightarrow Z_{Amutual} = 1.37\times 10^{-5} - j3.71\times 10^{-4}\ \Omega / m$$

Earth series self impedance *:

$$Z_{earth} = \frac{\rho_e m_e}{2\pi}\left[K_0\left(m_e R_4\right) + \frac{2}{4+m_e^2 R_4^2} e^{-2hm_e}\right]$$

$$\Leftrightarrow Z_{earth} = \frac{100\left(1.4+j1.4\right)\times 10^{-3}}{2\pi}\left[K_0\left(\left(1.4+j1.4\right)\times 10^{-3}\cdot 47.5\times 10^{-3}\right) + \frac{2}{4+\left(\left(1.4+j1.4\right)\times 10^{-3}\right)^2\left(47.5\times 10^{-3}\right)^2} e^{-2\cdot 1.1\left(1.4+j1.4\right)\times 10^{-3}}\right]$$

$$\Leftrightarrow Z_{earth} = 4.95\times 10^{-5} + j6.12\times 10^{-4}\ \Omega / m$$

Mutual earth impedance *:

$$Z_{earth\_mutual} = \frac{\rho_e m_e^2}{2\pi}\left[K_0\left(m_e d\right) + \frac{2}{4+m_e^2 d^2} e^{-2hm_e}\right]$$

$$\Leftrightarrow Z_{earth\_mutual} = \frac{100\left(\left(1.4+j1.4\right)\times 10^{-3}\right)^2}{2\pi}\left[K_0\left(\left(1.4+j1.4\right)\times 10^{-3}\cdot 0.109\right) + \frac{2}{4+\left(\left(1.4+j1.4\right)\times 10^{-3}\right)^2\left(0.109\right)^2} e^{-2\cdot 1.1\left(1.4+j1.4\right)\times 10^{-3}}\right]$$

$$\Leftrightarrow Z_{earth\_mutual} = 4.95\times 10^{-5} + j5.69\times 10^{-4}\ \Omega / m$$

*These values would have small differneces from phase to phase, but for simplicity we consider them equal

Z_{Ainner}=1.36x10^{-3}+j1.45x10^{-3} Ω/m
Z_{Aouter}=1.29x10^{-3}+j1.31x10^{-3} Ω/m
$Z_{AGinsul}$=0+j2.349x10^{-6} Ω/m
$Z_{Amutual}$=1.37x10^{-5}-j3.71x10^{-4} Ω/m
Z_{earth}≈4.95x10^{-5}+j6.12x10^{-4} Ω/m
Z_{earth_mutual}≈4.95x10^{-5}+j5.69x10^{-4} Ω/m

We can now write the loop matrix:

$$\left[Z_L\right] = \begin{bmatrix} Z_{L\_CS} & Z_{Lm\_CS} & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ Z_{Lm\_CS} & Z_{L\_SA} & Z_{Lm\_SA} & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & Z_{Lm\_SA} & Z_{L\_AG} & 0 & 0 & Z_{gm12} & 0 & 0 & Z_{gm13} \\ 0 & 0 & 0 & Z_{L\_CS} & Z_{Lm\_CS} & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & Z_{Lm\_CS} & Z_{L\_SA} & Z_{Lm\_SA} & 0 & 0 & 0 \\ 0 & 0 & Z_{gm12} & 0 & Z_{Lm\_SA} & Z_{L\_AG} & 0 & 0 & Z_{gm23} \\ 0 & 0 & 0 & 0 & 0 & 0 & Z_{L\_CS} & Z_{Lm\_CS} & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & Z_{Lm\_CS} & Z_{L\_SA} & Z_{Lm\_SA} \\ 0 & 0 & Z_{gm13} & 0 & 0 & Z_{gm23} & 0 & Z_{Lm\_SA} & Z_{L\_AG} \end{bmatrix}$$

Where: $Z_{L\_CS} = Z_{Couter} + Z_{CSinsul} + Z_{Sinner}$

$$Z_{L\_SA} = Z_{Souter} + Z_{SAinsul} + Z_{Ainner}$$

$$Z_{L\_AG} = Z_{Aouter} + Z_{AGinsul} + Z_{earth}$$

$$Z_{Lm\_CS} = -Z_{Smutual}$$

$$Z_{Lm\_SA} = -Z_{Amutual}$$

$$
[Z_L]=\begin{bmatrix}
0.2058+j0.0582 & -0.179+j0.0005 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0.2058+j0.058 & 1.5400+j1.4528 & -0.0137+j0.3713 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & -0.0137+j0.3713 & 1.3360+j1.9222 & 0 & 0 & 0.0495+j0.5687 & 0 & 0 & 0.0495+j0.5687 \\
0 & 0 & 0 & 0.2058+j0.058 & 0.2058+j0.058 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0.2058+j0.058 & 1.5400+j1.4528 & -0.0137+j0.3713 & 0 & 0 & 0 \\
0 & 0 & 0.0495+j0.5687 & 0 & -0.0137+j0.3713 & -0.0137+j0.3713 & 0 & 0 & 0.0495+j0.5687 \\
0 & 0 & 0 & 0 & 0 & 0 & 0.2058+j0.058 & 0.2058+j0.058 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0.2058+j0.058 & 1.5400+j1.4528 & -0.0137+j0.3713 \\
0 & 0 & 0.0495+j0.5687 & 0 & 0 & 0.0495+j0.5687 & 0 & -0.0137+j0.3713 & -0.0137+j0.3713
\end{bmatrix} m\Omega/m
$$

$$
[A]=\begin{bmatrix}
1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
-1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & -1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & -1 & 1 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & -1 & 1 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & -1 & 1 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 1
\end{bmatrix}
$$

$$
[Z]=\begin{bmatrix}
2.696+j4.177 & 2.670+j4.118 & 1.322+j2.294 & 0.050+j0.569 & 0.050+j0.569 & 0.050+j0.569 & 0.050+j0.569 & 0.050+j0.569 & 0.050+j0.569 \\
2.670+j4.118 & 2.849+j4.118 & 1.322+j2.294 & 0.050+j0.569 & 0.050+j0.569 & 0.050+j0.569 & 0.050+j0.569 & 0.050+j0.569 & 0.050+j0.569 \\
1.322+j2.294 & 1.322+j2.294 & 1.336+j1.922 & 0.050+j0.569 & 0.050+j0.569 & 0.050+j0.569 & 0.050+j0.569 & 0.050+j0.569 & 0.050+j0.569 \\
0.050+j0.569 & 0.050+j0.569 & 0.050+j0.569 & 2.696+j4.177 & 2.670+j4.118 & 1.322+j2.294 & 0.050+j0.569 & 0.050+j0.569 & 0.050+j0.569 \\
0.050+j0.569 & 0.050+j0.569 & 0.050+j0.569 & 2.670+j4.118 & 2.849+j4.118 & 1.322+j2.294 & 0.050+j0.569 & 0.050+j0.569 & 0.050+j0.569 \\
0.050+j0.569 & 0.050+j0.569 & 0.050+j0.569 & 1.322+j2.294 & 1.322+j2.294 & 1.336+j1.922 & 0.050+j0.569 & 0.050+j0.569 & 0.050+j0.569 \\
0.050+j0.569 & 0.050+j0.569 & 0.050+j0.569 & 0.050+j0.569 & 0.050+j0.569 & 0.050+j0.569 & 2.696+j4.177 & 2.670+j4.118 & 1.322+j2.294 \\
0.050+j0.569 & 0.050+j0.569 & 0.050+j0.569 & 0.050+j0.569 & 0.050+j0.569 & 0.050+j0.569 & 2.670+j4.118 & 2.849+j4.118 & 1.322+j2.294 \\
0.050+j0.569 & 0.050+j0.569 & 0.050+j0.569 & 0.050+j0.569 & 0.050+j0.569 & 0.050+j0.569 & 1.322+j2.294 & 1.322+j2.294 & 1.336+j1.922
\end{bmatrix} m\Omega/m
$$

4)
We use the code available online (note that the order of the elements in the matrices shown by the code may be different from the one shown below):

50Hz

$$[T_V] \simeq \begin{bmatrix} 0.404+j0.0405 & 0 & 0.042+j0.1536 & 0.5772 & 0.8055 & 0 \\ 0.404+j0.0405 & -0.0036+j0.1331 & -0.021-j0.0768 & 0.5772 & -0.4028 & 0.6976 \\ 0.404+j0.0405 & -0.0036-j0.1331 & -0.021-j0.0768 & 0.5772 & -0.4028 & -0.6976 \\ 0.4105 & 0 & 0.8019 & 0.002-j0.011 & 0.1316-j0.0211 & 0 \\ 0.4105 & 0.6945 & -0.401 & 0.002-j0.011 & -0.0658+j0.0106 & 0.114-j0.0183 \\ 0.4105 & -0.6945 & -0.401 & 0.002-j0.011 & -0.0658+j0.0106 & -0.114+j0.0183 \end{bmatrix}$$

$$[T_I] \simeq \begin{bmatrix} -0.0025-j0.0157 & 0 & -0.1372-j0.0176 & 0.5803+j0.0108 & 0.8317-j0.0261 & 0 \\ -0.0025-j0.0157 & -0.1188-j0.0152 & 0.0686+j0.0088 & 0.5803+j0.0108 & -0.4158+j0.013 & 0.7202-j0.0226 \\ -0.0025-j0.0157 & 0.1188+j0.0152 & 0.0686+j0.0088 & 0.5803+j0.0108 & -0.4158+j0.013 & -0.7202+j0.0226 \\ 0.8161+j0.0152 & 0 & 0.8354-j0.0262 & -0.5722+j0.0466 & 0+j0.1595 & 0 \\ 0.8161+j0.0152 & 0.7235-j0.0227 & -0.4177+j0.0131 & -0.5722+j0.046 & 0-j0.0797 & 0+j0.1381 \\ 0.8161+j0.0152 & -0.7235+j0.0227 & -0.4177+j0.0131 & -0.5722+j0.046 & 0-j0.0797 & 0-j0.1381 \end{bmatrix}$$

200Hz

$$[T_V] \simeq \begin{bmatrix} 0.4076+j0.011 & 0 & 0.2565+j0.3866 & 0.5773 & 0.8079 & 0 \\ 0.4076+j0.011 & -0.2222-j0.3349 & -0.1282-j0.1933 & 0.5773 & -0.4039 & -0.6997 \\ 0.4076+j0.011 & 0.2222+j0.3349 & -0.1282-j0.1933 & 0.5773 & -0.4039 & 0.6997 \\ 0.4087 & 0 & 0.6719 & 0-j0.003 & 0.0942-j0.0715 & 0 \\ 0.4087 & -0.5818 & -0.3359 & 0-j0.003 & -0.0471+j0.0358 & -0.0815+j0.062 \\ 0.4087 & 0.5818 & -0.3359 & 0-j0.003 & -0.0471+j0.0358 & 0.015-j0.062 \end{bmatrix}$$

$$[T_I] \simeq \begin{bmatrix} 0-j0.0043 & 0 & -0.1312-j0.0923 & 0.5777+j0.003 & 0.911-j0.0335 & 0 \\ 0-j0.0043 & 0.1137+j0.080 & 0.0656+j0.0461 & 0.5777+j0.003 & -0.4555+j0.0168 & 0.789-j0.029 \\ 0-j0.0043 & -0.1188-j0.080 & 0.0656+j0.461 & 0.5777+j0.003 & -0.4555+j0.0168 & -0.789+j0.029 \\ 0.8161+j0.0043 & 0 & 1.0954-j0.0403 & -0.5763+j0.0126 & -0.3285+j0.537 & 0 \\ 0.8161+j0.0043 & -0.9489+j0.0349 & -0.5477+j0.0201 & -0.5763+j0.0126 & 0.1643-j0.2685 & 0.2846-j0.4653 \\ 0.8161+j0.0043 & 0.9489-j0.0349 & -0.5477+j0.0201 & -0.5763+j0.0126 & 0.1643-j0.2685 & -0.2864+j0.4653 \end{bmatrix}$$

The results show that as the frequency increases the results get to be closer to decoupled matrices.

5)
The first step is to calculate the surge impedance and travelling times for both the cable and the OHL.

$$Z_{Cable} = \sqrt{\frac{L}{C}} \Leftrightarrow Z_{Cable} = \sqrt{\frac{0.4\times10^{-3}}{0.16\times10^{-6}}} \Leftrightarrow Z_{Cable} = 50\Omega$$

$$Z_{OHL} = \sqrt{\frac{L}{C}} \Leftrightarrow Z_{OHL} = \sqrt{\frac{1.6\times10^{-3}}{10\times10^{-9}}} \Leftrightarrow Z_{Cable} = 400\Omega$$

$$t_{Cable} = \frac{10}{\frac{1}{\sqrt{LC}}} \Leftrightarrow t_{Cable} = \frac{10}{\frac{1}{\sqrt{0.4\times10^{-3}\cdot 0.16\times10^{-6}}}} \Leftrightarrow t_{Cable} = 80\mu s$$

$$t_{OHL} = \frac{10}{\frac{1}{\sqrt{LC}}} \Leftrightarrow t_{OHL} = \frac{6}{\frac{1}{\sqrt{1.6\times10^{-3}\cdot 10\times10^{-9}}}} \Leftrightarrow t_{Cable} = 24\mu s$$

We can now calculate the reflection and refraction coefficients.

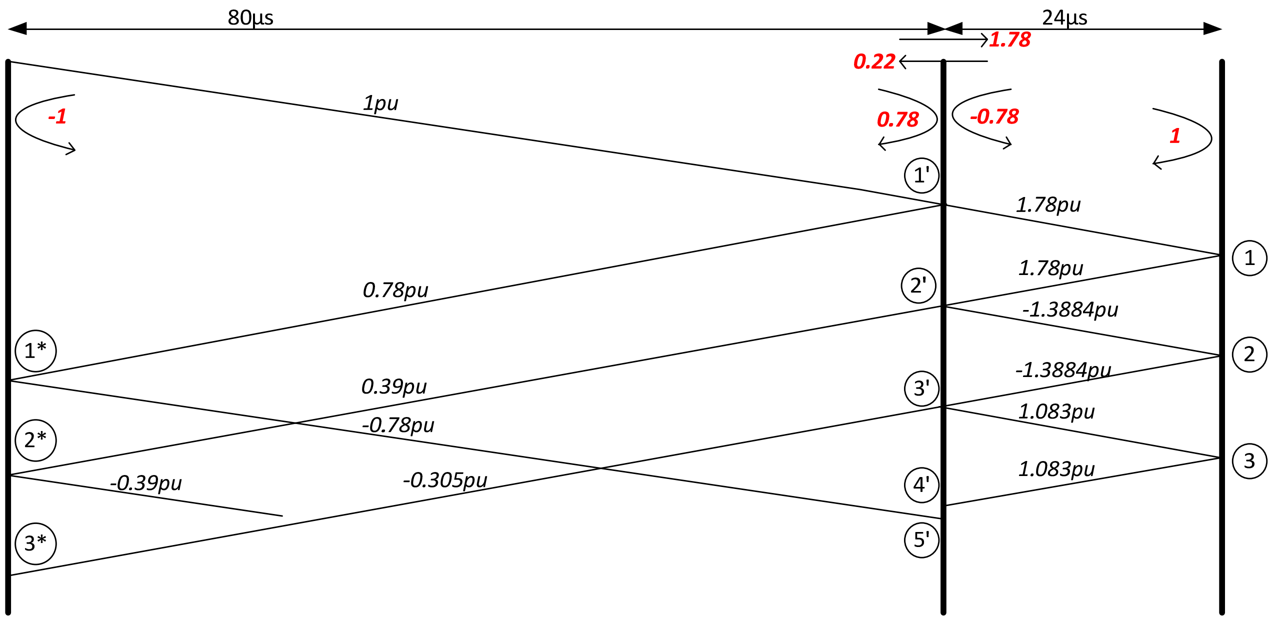
$$K_{Cable-OHL} = \frac{2Z_{OHL}}{Z_{Cable}+Z_{OHL}} \Leftrightarrow K_{Cable-OHL} = \frac{2\cdot 400}{50+400} \Leftrightarrow K_{Cable-OHL} = 1.78$$

$$K_{OHL-Cable} = \frac{2Z_{Cable}}{Z_{OHL}+Z_{Cable}} \Leftrightarrow K_{OHL-Cable} = \frac{2\cdot 50}{400+50} \Leftrightarrow K_{OHL-Cable} = 0.22$$

$$K_{Cable-Cable} = \frac{Z_{OHL}-Z_{Cable}}{Z_{Cable}+Z_{OHL}} \Leftrightarrow K_{Cable-Cable} = \frac{400-50}{50+400} \Leftrightarrow K_{Cable-Cable} = 0.78$$

$$K_{OHL-OHL} = \frac{Z_{Cable}-Z_{OHL}}{Z_{Cable}+Z_{OHL}} \Leftrightarrow K_{OHL-OHL} = \frac{50-400}{400+50} \Leftrightarrow K_{OHL-OHL} = -0.78$$

Considering the injection of a 1pu wave in the cable+OHL system, we can design the lattice diagram:



It is shown below the time at which the wave reaches the receiving end of the line. It is also shown the voltage in that point immediately after that instant.

1: (104µs): 1.78+1.78=3.56pu
2: (152µs): 3.56+2x(-1.3884)=0.783pu
3: (200µs): 0.783+2x(1.083)=2.949pu

It is shown below the time at which the wave reaches the sending end of the line. It is also shown the voltage in that point immediately after that instant. As we consider the line connected to an ideal voltage source, the voltage is constant and equal to 1pu at all instant (we also consider that the transient is so fast that the magnitude voltage at source does not change during its duration)

1*: (160μs): 1+0.78-0.78=1pu
2*: (208μs): 1+0.39-0.39=1pu
3*: (256μs): 1-0.305+0.305=1pu

It is shown below the time at which the wave reaches the joint point of the line. It is also shown the voltage in that point immediately after that instant on the cable and OHL side of the line. As expected, both voltages are equal.
1': (80μs): 1+0.78=1.78pu / 1.78pu
2': (128μs): 1.78+0.39=2.17pu / 1.78+1.78-1.3884=2.17pu
3': (176μs): 2.17-0.305=1.865pu / 2.1713-1.3884+1.083=1.865pu